Impact parameter dependence of parton densities at low x

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We consider impact parameter dependence of the polarized and unpolarized parton densities. Unitarity does not allow factorization of these structure functions over the Bjorken x and the impact parameter b variables. On the basis of the particular geometrical model approach, we conclude that spin of constituent quark may have a significant orbital angular momentum component which can manifest itself through the peripherality of the spin dependent structure functions.

The particular role of the low Bjorken x region in DIS is well known, it is this region where asymptotical properties of the strong interactions can be studied. The characteristic property of this region is an essential contribution of the nonperturbative dynamics [1,2] and one of the possible ways to treat this dynamics is the construction and application of models. Of course, the shortcomings of any model approach to the study of the nonperturbative dynamics are evident. However, one could hope to gain from a model approach an information which cannot currently be obtained otherwise, e.g. by the lattice QCD methods. The renewed topic in the subject of structure functions at low x is the study of their geometrical features, i.e. the considerations of the dependence of the parton densities on the transverse coordinates or the impact parameter. It has been shown [3] that the b-dependent parton distributions are related to the Fourier transform of the off-forward matrix elements of parton correlation functions in the limiting case of zero skewedness. Earlier connection of the form factors with the b-dependent distributions was considered in [4] and its generalization for the off-mass-shell case in [5].

The study of the impact parameter dependence allows one to gain information on the spatial distribution of the partons inside the parent hadron and on the spin properties of the nonperturbative intrinsic hadron structure. The geometrical properties of structure functions play also an important role in the analysis of the lepton–nuclei deep–inelastic scattering and in the hard production in heavy–ion collisions.

To consider geometrical features of parton densities we suppose that the DIS process is determined by the aligned-jet mechanism [1]. The aligned-jet mechanism is essentially nonperturbative. It allows one to relate structure functions with the discontinuities of the amplitudes of quark–hadron elastic scattering when an incoming quark is a hadronlike object which is close to its mass shell. These relations have the form [6]

$$q(x,Q^2) = \frac{1}{2}\mathrm{Im}[F_1(s,t,Q^2)+F_3(s,t,Q^2)]|_{t=0}, \quad (1)$$

$$\Delta q(x,Q^2) = \frac{1}{2}\mathrm{Im}[F_3(s,t,Q^2)-F_1(s,t,Q^2)]|_{t=0}, \quad (2)$$

$$\delta q(x,Q^2) = \frac{1}{2}\mathrm{Im}F_2(s,t,Q^2)|_{t=0}. \quad (3)$$

The functions F_i are helicity amplitudes for the elastic quark-hadron scattering in the standard notations for the nucleon–nucleon scattering. It should be noted that the structure functions obtained according to the above formulas should be multiplied by the factor $\sim 1/Q^2$ — probability that the virtual photon converts into the asymmetric small-$p_\perp$ quark-antiquark pair [1]. The quark virtuality is connected to the photon virtuality Q^2 and this is reflected by the dependence of the functions F_i on Q^2. The amplitudes $F_i(s,t,Q^2)$ are the corresponding Fourier-Bessel

transforms of the functions $F_i(s,b,Q^2)$. The relations (1-3) will be used as a starting point for the definition of the impact parameter dependent structure functions, i.e. it is natural to give the following operational definition:

$$q(x,b,Q^2) \equiv \frac{1}{2}\mathrm{Im}[F_1(x,b,Q^2)+F_3(x,b,Q^2)], \quad (4)$$

$$\Delta q(x,b,Q^2) \equiv \frac{1}{2}\mathrm{Im}[F_3(x,b,Q^2)-F_1(x,b,Q^2)], \quad (5)$$

$$\delta q(x,b,Q^2) \equiv \frac{1}{2}\mathrm{Im}F_2(x,b,Q^2), \quad (6)$$

and $q(x,Q^2)$, $\Delta q(x,Q^2)$ and $\delta q(x,Q^2)$ are the integrals over b of the corresponding b-dependent distributions, i.e.

$$q(x,Q^2) = \frac{Q^2}{\pi^2 x}\int_0^\infty db\, b\, q(x,b,Q^2), \quad (7)$$

$$\Delta q(x,Q^2) = \frac{Q^2}{\pi^2 x}\int_0^\infty db\, b\, \Delta q(x,b,Q^2), \quad (8)$$

$$\delta q(x,Q^2) = \frac{Q^2}{\pi^2 x}\int_0^\infty db\, b\, \delta q(x,b,Q^2). \quad (9)$$

The functions $q(x,b,Q^2)$, $\Delta q(x,b,Q^2)$ and $\delta q(x,b,Q^2)$ have simple interpretation, e.g. the function $q(x,b,Q^2)$ is a number density of quarks q with fraction x of the hadron longitudinal momentum at the transverse distance b from the hadron geometrical center. It should be noted that unitarity plays crucial role in the probabilistic interpretation of the function $q(x,b,Q^2)$. Indeed due to unitarity

$$0 \le q(x,b,Q^2) \le 1.$$

The integrated distribution $q(x,Q^2)$ is not limited by unity and can have arbitrary non–negative value.

Interpretation of the spin distributions directly follows from their definitions: they are the differences of the corresponding spin dependent quark number densities.

Unitarity can be fulfilled through the U–matrix representation for the helicity amplitudes of elastic quark–hadron scattering. In the impact parameter representation the expressions for the helicity amplitudes are the following

$$F_{1,3}(x,b,Q^2) = \frac{U_{1,3}(x,b,Q^2)}{[1-iU_{1,3}(x,b,Q^2)]}, \quad (10)$$

$$F_2(x,b,Q^2) = \frac{U_2(x,b,Q^2)}{[1-iU_1(x,b,Q^2)]^2}. \quad (11)$$

Unitarity requires Im $U_{1,3}(x,b,Q^2) \ge 0$. It is to be noted that the U–matrix form of the unitary representation, contrary to the eikonal one, does not generate essential singularity in the complex x plane at $x \to 0$.

We consider the structure functions along the lines outlined in [7]. A hadron consists of the constituent quarks aligned in the longitudinal direction and embedded into the nonperturbative vacuum (condensate). The constituent quark appears as a quasiparticle, i.e. as current valence quark surrounded by the cloud of quark-antiquark pairs of different flavors. We refer to effective QCD approach and use the Nambu–Jona-Lasinio (NJL) model as a basis. The Lagrangian in addition to the four–fermion interaction $\mathcal{L}_4$ of the original NJL model includes the six–fermion $U(1)_A$–breaking term $\mathcal{L}_6 \propto K(\bar{u}u)(\bar{d}d)(\bar{s}s)$. Transition to partonic picture is described by the introduction of a momentum cutoff $\Lambda = \Lambda_\chi \simeq 1$ GeV, which corresponds to the scale of chiral symmetry spontaneous breaking. This picture for a hadron structure implies that overlapping and interaction of peripheral condensates in hadron collision occurs at the first stage. In the overlapping region the condensates interact and as a result virtual massive quark pairs appear. A part of hadron energy carried by the peripheral condensates goes to generation of massive quarks. In other words nonlinear field couplings transform kinetic energy into internal energy of dressed quarks. Of course, number of such quarks fluctuates. The average number of quarks in the considered case is proportional to the convolution of the condensate distributions $D_c^{Q,H}$ of the colliding constituent quark and hadron:

$$N(s,b,Q^2) \simeq N(s,Q^2)\cdot D_c^Q \otimes D_c^H, \quad (12)$$

where the function $N(s)$ is determined by a transformation thermodynamics of kinetic energy of

interacting condensates to the internal energy of massive quarks. To estimate the $N(s,Q^2)$ it is feasible to assume that it is proportional to the maximal possible energy dependence

$$N(s,Q^2) \simeq \kappa(Q^2)(1-\langle x_Q\rangle)\sqrt{s}/\langle m_Q\rangle, \quad (13)$$

where $\langle x_Q\rangle$ is the average fraction of energy carried by the constituent quarks, $\langle m_Q\rangle$ is the mass scale of constituent quarks. In the model each of the constituent valence quarks located in the central part of the hadron is supposed to scatter in a quasi-independent way by the generated virtual quark pairs at given impact parameter and by the other valence quarks. The strong interaction radius of the constituent quark Q is determined by its Compton wavelength and the b–dependence of the function $\langle f_Q\rangle$ related to the quark form factor $F_Q(q^2)$ has a simple form $\langle f_Q\rangle \propto \exp(-m_Q b/\xi)$. The helicity flip transition, i.e. $Q_+ \to Q_-$ occurs when the valence quark knocks out a quark with the opposite helicity and the same flavor. The helicity functions $U_i(x,b,Q^2)$ at small values of x have the following dependence [7]:

$$\begin{aligned} U_{1,3}(x,b,Q^2) &= c_{1,3}(x,Q^2)U_0(x,b,Q^2), \quad (14)\\ U_2(x,b,Q^2) &= d(x,b,Q^2)U_0(x,b,Q^2), \quad (15)\end{aligned}$$

where

$$c_{1,3}(x,Q^2) = 1+\beta_{1,3}(Q^2)m_Q\sqrt{x}/Q,$$

$$d(x,b,Q^2) = \frac{g_f^2(Q^2)m_Q^2 x}{Q^2}\exp[-2(\alpha-1)m_Q b/\xi]$$

and

$$\begin{aligned} U_0(x,b,Q^2) &= i\tilde{U}_0(x,b,Q^2)\\ &= i\left[\frac{a(Q^2)Q}{m_Q\sqrt{x}}\right]^N \exp[-Mb/\xi]. \quad (16)\end{aligned}$$

a, α, β, g_f and ξ are the model parameters, some of them depend on the virtuality Q^2.

Then using Eqs. (10,11) we obtain at small x:

$$q(x,b,Q^2) = \frac{\tilde{U}_0(x,b,Q^2)}{1+\tilde{U}_0(x,b,Q^2)}, \quad (17)$$

$$\Delta q(x,b,Q^2) = \frac{c_-(x,Q^2)}{2}\frac{\tilde{U}_0(x,b,Q^2)}{[1+\tilde{U}_0(x,b,Q^2)]^2}, \quad (18)$$

$$\delta q(x,b,Q^2) = \frac{d(x,b,Q^2)}{2}\frac{\tilde{U}_0(x,b,Q^2)}{[1+\tilde{U}_0(x,b,Q^2)]^2}, \quad (19)$$

where $c_-(x,Q^2) = c_3(x,Q^2) - c_1(x,Q^2)$.

From the above expressions it follows that $q(x,b,Q^2)$ has a central b–dependence, while $\Delta q(x,b,Q^2)$ and $\delta q(x,b,Q^2)$ have peripheral profiles.

From Eqs. (17-19) it follows that factorization of x and b dependencies is not allowed by unitarity and this provides certain constrains for the model parameterizations of structure functions. Indeed, it is clear from Eqs. (17-19) that factorized form of the input "amplitude" $\tilde{U}_0(x,b,Q^2)$ cannot survive after unitarization due to the presence of the denominators. It is to be noted here that from the relation of impact parameter distributions with the off-forward parton distributions [3] it follows that the same conclusion on the absence of factorization is also valid for the off-forward parton distributions with zero skewedness.

It is interesting to note that the spin structure functions have a peripheral dependence on the impact parameter contrary to the central profile of the unpolarized structure function. It could be related with the orbital angular momentum of quark pairs inside the constituent quark. The important point is what the origin of this orbital angular momenta is. It was proposed to use an analogy with an anisotropic extension of the theory of superconductivity which seems to match well with the picture for a constituent quark. The studies of that theory show that the presence of anisotropy leads to axial symmetry of pairing correlations around the anisotropy direction $\hat{\vec{l}}$ and to the particle currents induced by the pairing correlations. In other words it means that a particle of the condensed fluid is surrounded by a cloud of correlated particles which rotate around it with the axis of rotation $\hat{\vec{l}}$. Calculation of the orbital momentum shows that it is proportional to the density of the correlated particles. Thus, it is clear that there is a direct analogy between this picture and the one describing the constituent quark. An axis of anisotropy $\hat{\vec{l}}$ can be associated with the polarization vector of current

valence quark located at the origin of the constituent quark. The orbital angular momentum $\vec{L}$ lies along $\hat{\vec{l}}$.

The spin of a constituent quark, e.g. U-quark, in the used model is given by the sum:

$$\begin{aligned} J_U = 1/2 &= S_{u_v} + S_{\{\bar{q}q\}} + L_{\{\bar{q}q\}} \\ &= 1/2 + S_{\{\bar{q}q\}} + L_{\{\bar{q}q\}}. \end{aligned} \quad (20)$$

In principle, $S_{\{\bar{q}q\}}$ and $L_{\{\bar{q}q\}}$ can include contribution of gluon angular momentum. However, since we consider effective Lagrangian approach where gluon degrees of freedom are overintegrated, we do not touch the problems of the separation and mixing of the quark angular momentum and gluon effects in QCD. Indeed, in the extension of the NJL–model the six-quark fermion operator simulates the effect of gluon operator $\frac{\alpha_s}{2\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$, where $G_{\mu\nu}$ is the gluon field tensor in QCD.

The value of the orbital momentum contribution into the spin of constituent quark can be estimated according to the relation between contributions of current quarks into a proton spin and corresponding contributions of current quarks into a spin of the constituent quarks and that of the constituent quarks into proton spin:

$$(\Delta\Sigma)_p = (\Delta U + \Delta D)(\Delta\Sigma)_U, \quad (21)$$

where $(\Delta\Sigma)_U = S_{u_v} + S_{\{\bar{q}q\}}$. The value of $(\Delta\Sigma)_p$ was measured in the deep–inelastic scattering. Thus, on the grounds of the experimental data for polarized DIS we arrive to conclusion that the significant part of the spin of constituent quark should be associated with the orbital angular momentum of the current quarks inside the constituent one.

Then the peripherality of the spin structure functions can be correlated with the large contribution of the orbital angular momentum, i.e. with the quarks coherent rotation. Indeed, there is a compensation between the total spin of the quark-antiquark cloud and its orbital angular momenta, i.e. $L_{\{\bar{q}q\}} = -S_{\{\bar{q}q\}}$ and therefore the above correlation follows if such compensation has a local nature and valid for a fixed impact parameter.

The important role of orbital angular momentum was known [8] before the European Muon Collaboration at CERN found that only small fraction of a nucleon spin is due to the quark contribution and reappeared later as one of the transparent explanations of the polarized deep-inelastic scattering data [9]. Lattice QCD calculations in the quenched approximation also indicate significant quark orbital angular momentum contribution to spin of a nucleon [10]. The issue of the orbital angular momentum is important for the explanation of polarization effects in the hyperon productions [11].

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REFERENCES

1. J. Bjorken, AIP Conference Proceedings No. 6, Particles and Fields Subseries, No. 2, ed. M. Bander, G. Shaw and D. Wang (AIP, New York, 1972), SLAC-PUB-7096, 1996.
2. E. A. Paschos, Phys. Lett. B 389 (1996) 383.
3. M. Burkardt, Phys. Rev. D62 (2000) 71503; hep-ph/0005108; hep-ph/0008051.
4. T. T. Chou and C. N. Yang, Phys. Rev. 170 (1968) 1591.
5. V. A. Petrov, Nucl. Phys. Proc. Suppl. 54A (1997) 160.
6. R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552.
7. S. M. Troshin and N. E. Tyurin, Phys. Rev. D57 (1998) 5447.
8. L. M. Sehgal, Phys. Rev. D10 (1974) 1663.
9. S. J. Brodsky, J. Ellis, M. Karliner, Phys. Lett. B206 (1988) 309.
10. N. Mathur, S. J. Dong, K. F. Liu, L. Mankiewicz, and N. C. Mukhopadhyay, hep-ph/9912289.
11. S.M. Troshin and N.E. Tyurin, Phys.Rev. D55 (1997) 1265.